Trusses

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Abstract

In the 1920s H. Prüfer [3, page 170] and R. Baer [1, page 202] defined *a heap* as an algebraic system $(H, [-,-,-])$ consisting of a nonempty set H and a ternary operation $[-,-,-]\colon H \times H \times H \to H$, $(x, y, z) \mapsto [x, y, z]$ satisfying associativity $[[x, y, z], t, u] = [x, y, [z, t, u]]$ and Mal'cev identities $[x, x, y] = y = [y, x, x]$ for all $x, y, z, t, u \in H$. A heap $(H, [-,-,-])$ is said to be abelian, if $[x, y, z] = [z, y, x]$ for all $x, y, z \in H$. Starting with any heap $(H, [-,-,-])$ we can assign a group $(H, \circ_e, e)$ to it by fixing the middle entry of the ternary operation $[-,-,-]$, that is for an arbitrary but fixed element $e \in H$, setting $x \circ_e y := [x, e, y]$ for all $x, y \in H$, we obtain a group operation on H. Conversely, every group $(G, \circ, 1)$ gives rise to a heap $(G, [-,-,-]_\circ)$ by taking the ternary operation $[x, y, z]_\circ := x \circ y^{-1} \circ z$ for all $x, y, z \in G$. A heap can be understood as a group in which the neutral element has not been specified. A choice of any element in a heap can reduce the ternary operation to a binary operation that makes the underlying set into a group in which the chosen element is the neutral element. Enriching a heap with associative binary operation which distributes over the ternary heap operation is a natural progression that mimics the process which leads from groups to rings. In 2019, T. Brzeziński [2] defined *a truss* as an algebraic system $(T, [-,-,-,], \cdot)$ consisting of a nonempty set T, a ternary operation $[-,-,-]$ making T into an abelian heap, and a binary operation $\cdot$ making T into and semigroup and satisfying distributivity $x \cdot [y, z, t] = [x \cdot y, x \cdot z, x \cdot t]$ and $[x, y, z] \cdot t = [x \cdot t, y \cdot t, z \cdot t]$ for all $x, y, z \in T$. This talk is intended as a discussion of trusses.

Keywords

Heaps, Braces, Trusses.

References

[1] R. Baer, *Zur Einführung des Scharbegriffs*, Journal für die Reine und Angewandte Mathematik 160 (1929), 199-207.

[2] T. Brzeziński, *Trusses: Between braces and rings*, Transactions of the American Mathematical Society 372 (2019), 4149–4176.

[3] H. Prüfer, *Theorie der Abelschen Gruppen. I. Grundeigenschaften*, Mathematische Zeitschrift 20 (1924), 165-187.